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SHOW that when the ellipse varies subject to the condition, $a + b$ constant, the locus of T is the hypocycloid,

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = (a + b)^{\frac{2}{3}},$$

which touches each ellipse at the point T.

SOLUTION.

The equation of the ellipse is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, \qquad (1)$

and we have $\qquad a + b = c = \text{a constant}. \qquad (2)$

Differentiating (1) and (2), a and b only variable,

$$\frac{x^2}{a^2}\,da = -\frac{ay^2}{b^3}\,db, \tag{3}$$

$$da = -\,db\,; \tag{4}$$

$$\therefore \quad \frac{x^2}{a^2} = \frac{ay^2}{b^3}. \tag{5}$$

$$\therefore \quad (1) \text{ is } \quad \frac{y^2}{b^3}(a + b) = 1, \quad \text{or} \quad b = c^{\frac{1}{3}}y^{\frac{2}{3}}, \quad \text{and} \quad a = c^{\frac{1}{3}}x^{\frac{2}{3}}.$$

These in (2) give $x^{\frac{2}{3}} + y^{\frac{2}{3}} = (a + b)^{\frac{2}{3}}$, a hypocycloid. From this equation $\dfrac{dy}{dx} = -\dfrac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$, which for $x = x' = \dfrac{a}{\sqrt{(2 - a)}}$, $y = y'' = \dfrac{a\,(1 - a)}{\sqrt{(2 - a)}}$, becomes $-\sqrt{(1 - a)}$, showing that the ellipse and hypocycloid have a common tangent at T. [*William Hoover.*]

EXERCISES.

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INTEGRATE the differential,

$$\frac{\sin\theta\cos\theta}{\sin^4\theta + \cos^4\theta}\,d\theta.$$

[*O. Root, Jr.*]

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ABOUT the vertices of an equilateral triangle three spheres are drawn with radii equal to the side of the triangle. Find the volume common to them all.

[*W. M. Thornton.*]

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An ellipse touches the given ellipse $a^2y^2 + b^2x^2 - a^2b^2$ at the extremities of a diameter and has its foci on the curve. Find the position of its axis major, and the position of the foci when the angle between the given diameter and the axis major of the given ellipse is a maximum. [*R. H. Graves.*]

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In the triangle ABC, the radius of the inscribed circle is

$$\frac{1}{3}\,\frac{(b + c)\cos^2\tfrac{1}{2}A + (c + a)\cos^2\tfrac{1}{2}B + (a + b)\cos^2\tfrac{1}{2}C}{\cot\tfrac{1}{2}A + \cot\tfrac{1}{2}B + \cot\tfrac{1}{2}C}.$$

[*Ormond Stone.*]

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Solve the differential equation,

$$u d^2y + 2\,du\,dy = v\,dx\,dy,$$

u, v, y being all functions of x. [*William Hoover.*]

142

Let p be a prime number of the form $4n - 1$, n being an integer. Let the primitive p^{th} roots of unity be

$$w,\ w^{\lambda},\ w^{\lambda^2},\ \ldots\ w^{\lambda^{p-2}}.$$

Put $\theta = 2\pi / p$, and let the negative terms of the series,

$$\sin\theta,\quad \sin(\lambda^2\theta),\quad \sin(\lambda^4\theta),\ \ldots\ \sin(\lambda^{p-3}\theta),$$

be $\sigma', \sigma'', \sigma''', \ldots$ Then

$$p = [\tan\tfrac{1}{4}(p + 1)\theta - 4(\sigma' + \sigma'' + \sigma''' + \ldots)]^2.$$

Verify this relation for the particular cases $p = 3, 7, 11$, and give a general proof. [*George Paxton Young.*]

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If a curve of the fourth order have a singular point at which are three coincident tangents, this line passes through the intersection point of the bitangent with the line of junction of the two points of inflexion of the curve.

[*F. H. Loud.*]

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Show that the diurnal path of the shadow of the top of a vertical rod on a horizontal plane will be a rectangular hyperbola, if

$$2\sin\delta = \sin\left(\frac{\pi}{4} + \varphi\right) + \sin\left(\frac{\pi}{4} - \varphi\right),$$

where φ is the latitude of the place and δ the declination of the sun, assumed to be constant. [*R. H. Graves.*]